Algebra II
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Lesson 13 - Locus
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1. The Real Line

The set of all real numbers is denoted $\mathbb{R}$. That is,

$$\mathbb{R} = \{x \mid x \text{ has a decimal expansion }\}.$$

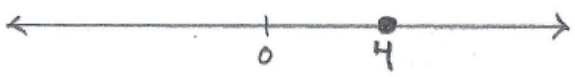
Geometrically, we view $\mathbb{R}$ as a line.

2. Locus in One Variable

The *locus* of an equation or inequality in one variable (say x) is the set of all real numbers, which, when plugged into the equation, make it true. For most purposes, the word "locus" is synonymous with "solution set".

Example 1. Find the locus of the equation $3x - 8 = 4$. Sketch this set.

Solution. Add 8 to both sides to get $3x = 12$. Divide both sides by 3 to get $x = 4$. The locus is $\{4\}$.



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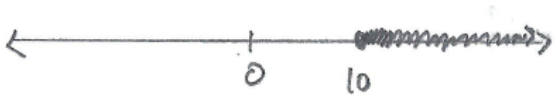
Example 2. Find the locus of the equation $5x - 8 \geq 3x + 12$. Sketch this set.

Solution. Add 8 to both sides to get $5x \geq 3x + 20$. Subtract $3x$ from both sides to get $2x \geq 20$. Divide both sides by 2 to get $x \geq 10$. The locus is $\{x \in \mathbb{R} \mid x \geq 10\}$.



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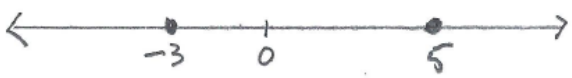
Example 3. Find the locus of the equation $x^2 - 2x - 15 = 0$. Sketch this set.

Solution. Factor to get $(x + 3)(x - 5) = 0$. Thus $x + 3 = 0$ or $x - 5 = 0$. Therefore $x = -3$ or $x = 5$. The locus is $\{-3, 5\}$.



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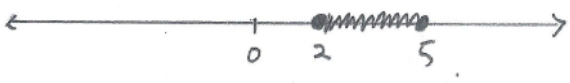
Example 4. Find the locus of the equation $x^2 - 7x + 10 \leq 0$. Sketch this set.

Solution. Factor to get $(x - 2)(x - 5) \leq 0$. The left hand side is the product of two expressions. If the expressions have different signs, then the result is negative; otherwise it is positive. There are three cases:

- If $x > 5$, then $x - 2$ and $x - 5$ are both positive, so the product is positive.
- If $x < 2$, then $x - 2$ is negative and $x - 5$ is negative.
- If x between 2 and 5, $x - 2$ is positive and $x - 5$ is negative.

Thus the locus is $\{x \in \mathbb{R} \mid 2 \leq x \leq 5\}$.



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3. The Cartesian Plane

The set of all ordered pairs of real numbers is denoted $\mathbb{R}^2$. That is,

$$\mathbb{R}^2 = \{(x, y) \mid x, y \in \mathbb{R}\}.$$

Geometrically, we view $\mathbb{R}^2$ as a plane.

4. Locus in Two Variables

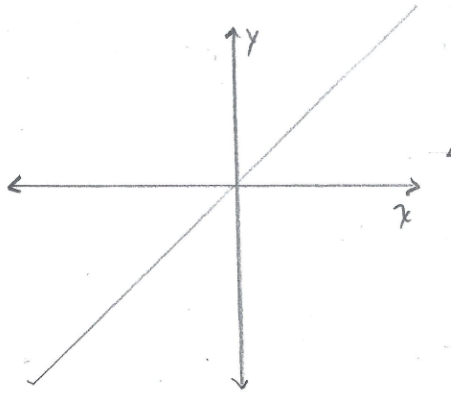
The *locus* of an equation or inequality in two variables (say x and y) is the set of all ordered pairs (x, y) of real numbers, which, when plugged into the equation, make it true.

Example 5. Find the locus of the equation $y = x$.

Solution. The locus is the set

$$\{(x, y) \in \mathbb{R}^2 \mid y = x\}.$$

The *graph* is a picture of this set in the cartesian plane; it is the diagonal line through the origin on which the x and y coordinates are equal.



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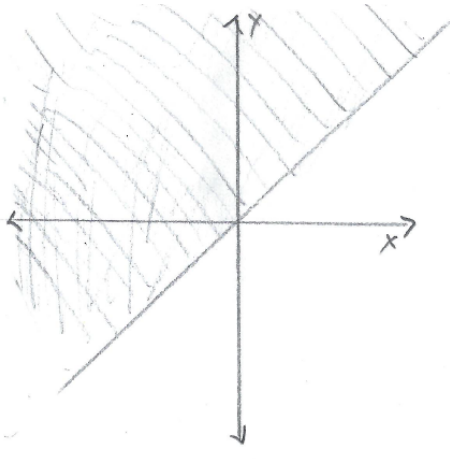
Example 6. Find the locus of the inequality $y \geq x$.

Solution. The locus is the set

$$\{(x, y) \in \mathbb{R}^2 \mid y \geq x\}.$$

The *graph* is a picture of this set in the cartesian plane; it is the shaded half plane above and including the diagonal line through the origin on which the x and y coordinates are equal.



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